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Effect of desired speed variability on highway traffic flow

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Traffic flow is a function of many natural, environmental, and human factors. Not only that weather and road condition can vary, but drivers' decisions and policies also can affect the flow. Here we analyze the effect of distribution of desired speeds. We show that a broader distribution can reduce the flow efficiency and increase congestions. Since different drivers react differently to changes in weather or road conditions, such a change leads to a change in desired speed distribution as well. As a result, nonintuitive changes in traffic flow may occur. Besides providing insight and analyzing the underlying mechanism of a collective phenomenon, this example sheds light on a fundamental aspect of computational modeling. Although "mean-field" models that deal with average values only and ignore variability are simpler and easier to analyze, they can very easily turn into oversimplifications and miss relevant qualitative phenomena.

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I. INTRODUCTION

Traffic problems have been modeled mathematically and computationally over the last two decades in various contexts. There is a large variety of models, for one-dimensional [1–3], two-dimensional [4,5], or network geometries [6–8], and for either single- [3,9] or multilane [10,11] roads. In many cases a nonuniform speed distribution has been assumed [12–14], and there are also studies about formation of variability and the resulting distribution [15,16]. Here we focus not on the actual speed distribution, but on the heterogeneity in drivers' policies that lead to that distribution. Different drivers may have different policies in determining the vehicle velocity in response to any given set of conditions. Thus, as road or weather conditions are changed, the desired speed distribution may be changed as well. This change affects the actual speeds and traffic flow. In addition, the desired speed reflects the expectation of the driver from the traffic flow under the given conditions. Change in these expectations may cause the driver to define the traffic flow as unsatisfactory, or congested.

Speed variability causes drivers to move more often between lanes. The effect of changing lanes on multilane traffic flow has been modeled and analyzed in the past [17–19]. It was shown that voids, caused by changing lanes, reduce the total traffic flow [19]. On the other hand, under realistic conditions, where not all vehicles run in the same speed, changing lanes is crucial to maintaining the flow. Without the option of changing lanes, the slowest vehicle would determine the speed for all other vehicles on the lane. Thus, changing lanes can either decrease or increase the average speed. Since the need for lane changing emerges from the heterogeneity in speeds, we wanted to examine systematically the effect of speed distribution on traffic flow. To this end we constructed a simple model of multilane road with distribution of vehicles' speed. In the next section we present our computational model in details. Results and analysis are presented in Sec. III. We conclude with a general discussion about modeling of complex systems in Sec. IV.

II. COMPUTATIONAL MODEL

Our model consists of continuous single- or multilane freeway. The initial condition was an empty highway, consists of L semi-infinite lanes. The dynamics is deterministic and the only stochastic dynamic component is the time delay between arrivals of new vehicles onto the freeway. Movement of a vehicle on the road is determined by simple rules: each vehicles has a desired velocity v_d. This is the velocity of the vehicle under ideal conditions, where there are no other vehicles on the road. The desired velocities are taken from a normal distribution [15,16,20] with mean V and standard deviation $\sigma = C_v \times V$, where C_v is the coefficient of variance. A list of desired velocities was taken from this distribution with given mean and standard deviation as indicated in each case. The negative velocities have been deleted and replaced by new velocities, taken from the same distribution. Presence of other vehicles in front of a running car may cause deceleration in order to prevent accidents. In case that the actual velocity, v_a, is slower than the desired velocity, and moving to a neighboring lane would improve the vehicle position, then the vehicle changes its lane.

A numerical code was written, using MATLAB™ [21]. New vehicles entered a lane at $x=0$, conditioned that last car in that lane was far enough so that there will be no accident. We denote by x_n^j the location of the nth vehicle on the jth lane. Then, a car with desired (and initial) velocity v_d would enter the jth lane if

$$\min\{x_n^j\} > v_d \Delta t,$$

where the minimum is taken over all vehicles in the lane (all n values) and Δt is the time step. Then a time gap to arrival of the next vehicle to that lane was randomly taken from uniform distribution between 0 and T (s). Unless otherwise indicated, we used $T=2$ s. Each of the iterations was composed of three steps: speed update, driving, and lane changes. In the first step, vehicles with $v_a < v_d$ whose distance to the next vehicle was large enough, increased their velocity. If the distance to next vehicle was greater than five vehicle lengths, the velocity was updated to v_d. In case of smaller distances, the increase was in a lesser extent. Namely, for vehicles which satisfy the condition,

$$v_a < v_d \text{ and } (x_{n+1}^j - x_n^j) > v_a \Delta t + l,$$

the velocity v_a was updated to

$$\min\left\{v_d, v_a + (v_d - v_a)\frac{(x^j_{n+1} - x^j_n)}{5l}\right\},$$

where l is the vehicle length. This acceleration step may assign vehicles velocities that can cause accident (in case the vehicle in front is too slow). This problem is taken care at the next speed update step, which is performed before the actual driving step. In this step, vehicles that were too close to the next vehicle reduced their speed to the exact velocity required to avoid accidents at the next time step. Namely, speed of vehicles that satisfy

$$v_a\Delta t + l > x^j_{n+1} - x^j_n$$

was changed to

$$v_a = (x^j_{n+1} - x^j_n - l)/\Delta t.$$

In the next step, all vehicles updated their location according to their respective velocities, $x_n = x_n + v_a(n)\Delta t$. At the bypassing step, there were two necessary conditions for a vehicle in order to change its lane: (1) running too slow, namely, $v_a < 95\% v_d$; and (2) having better position at a neighbor lane. Better position means either larger free space to the next vehicle or faster vehicle in front. All vehicles that fulfilled these two conditions changed their lanes. In order to avoid conflicts, at even time steps lane change was permitted only on one direction (from j to $j+1$), and in odd time steps bypassing to the other direction took place (from j to $j-1$). Each simulation ran for 1 h of traveling vehicles and then statistics was taken from all the vehicles presented on the road. The results have been averaged over 40 runs of the simulations.

III. RESULTS AND ANALYSIS

We define a satisfaction index η by calculating the ratio v_a/v_d, averaged over all running vehicles. $\eta = 1$ implies ideal flow where all vehicles are running at their desired speed. Lower values of η mean that there is a slow down of the traffic and vehicles are forced to run slower than they would like to. In other words, η is a measure of traffic congestion [22]. In the extreme case of $\sigma = 0$, all vehicles have the same desired velocity v_d. In this situation, there is no reason for any vehicle to slow down since the vehicle in front of any given vehicle does not run slower than the vehicle behind. Thus, in this situation $v_a = v_d$ for all vehicles and there is perfect satisfaction, namely, $\eta = 1$. Broader distribution of desired speeds is accompanied by higher variability in actual speeds. As the variability in speeds gets larger, there are more slow vehicles which force the cars behind them to run slower than they want to, and the satisfaction index decreases. In a single lane free way, there is no way to bypass the slow vehicles, and thus η decreases rapidly with C_v. This decrease is not dependent on the average velocity V but only on the width of the distribution [Fig. 1(A)].

Opening more lanes enables bypassing and improves the traffic flow significantly. Does this improvement depend on the speed distribution or on the average velocity? Numerical simulations show that the width of the distribution has a much more significant role than the absolute velocities [Fig. 1(B)]. Increase in the coefficient of variance causes a decrease in the efficiency of the traffic flow, and this decrease is similar for a broad range of mean velocities. Increasing the variance leads to two opposing effects: on the one hand, higher variability in velocities means more cases in which fast vehicles are significantly delayed by slower ones. Thus, as the variability increases, there is more need for bypassing and lane changing. On the other hand, more variability leads also to larger distances between successive vehicles, making the lane changing easier. Changes in the width of distribution can thus change the road dynamics and it is not surprising to see that the traffic flow is dependent on C_v. However, changing the mean velocity without changing C_v is mathematically equivalent to changing the time (or length) units. This is similar to taking a film of a road with high mean velocity and presenting it in slow motion. Obviously, if there is no congestion in the fast running, there cannot be either in the slow one.

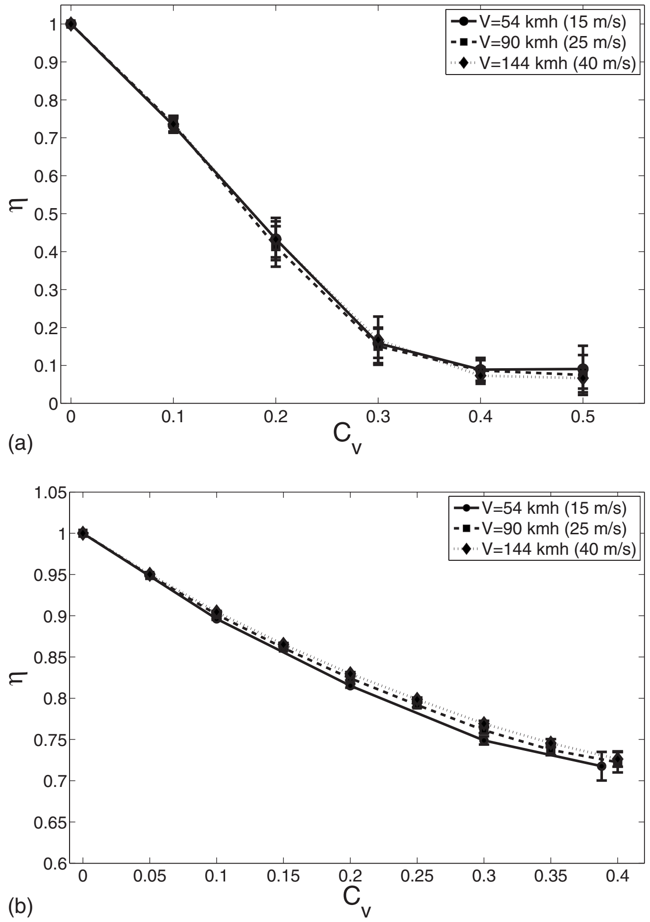

FIG. 1. The satisfaction index η depends on the variance coefficient but not on the mean velocity. (a) Effect of variance coefficient in case of a single lane. The satisfaction index decreases rapidly with variance. At lower values of C_v, by increasing C_v we have more slow vehicles (as well as fast ones). Since there are no negative velocities in this model, for $C_v > 0.4$ increasing the variance adds mainly fast vehicles. Since these vehicles are forced to run on a very slow velocity, the change in the satisfaction index is negligible. (b) In a five-lanes highway, bypassing and lane changing enable faster traffic flow. However, high variability still causes obstacles and reduces satisfaction.

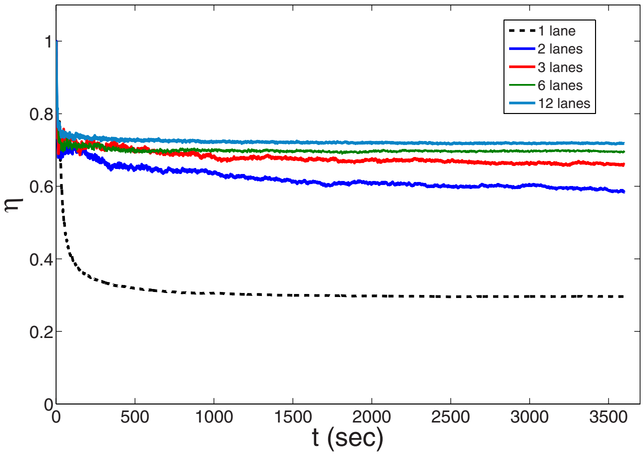

FIG. 2. (Color online) As the number of lanes increases, satisfaction index increases as well. However, the effect of addition lanes beyond the second is minor. The dashed line indicates a single lane, and the solid ones (from bottom up) are for 2, 3, 6, and 12 lanes. (Single simulation for each number of lanes, $C_v=0.25$.)

The satisfaction index η may be affected by the number of open lanes. One may hypothesize that more lanes will result in more efficient flow. To some extent, this is indeed the case. A significant improvement is observed by changing a single lane road into a two-lanes way. However, the contribution of any additional lane is minor (Fig. 2). Below we will show how this observation can be useful under certain circumstances.

Since in our model vehicles are added to each lane independently, the number of lanes does not change the global road density. However, vehicle density is a parameter of the system which can be tuned. This parameter may affect the traffic flow. As density increases, the average space between vehicles decreases and it becomes more difficult to change lanes. However, high density is the situation where lane change is mostly required in order to maintain high traffic flow. Thus, in addition to the distribution width, the total density is also a crucial parameter. In our model, the density is determined by the time gap between arrivals of new vehicles to the road. Very small time gaps are not feasible since a vehicle cannot enter the road if the car in front is too close. At the other extreme, a very large gap (and low density) implies that there is almost no interaction between vehicles and the actual velocity of any vehicles is the same as its desired speed. Between these limit cases there is an increase in the satisfaction index as function of the time gap T (Fig. 3).

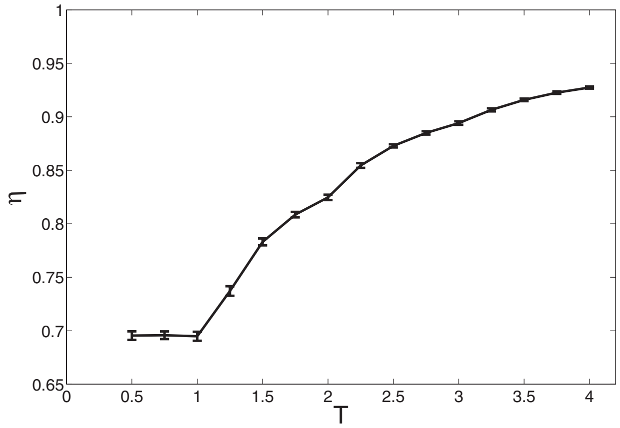

FIG. 3. Maximal time delay between arrivals of successive vehicles affects positively the satisfaction index. The actual time gap is taken randomly from uniform distribution between 0 and T.

Desired velocity is a function of many parameters like road condition, weather, density etc. Interestingly, not all drivers respond in a similar way to changes in the road conditions such as bad weather or traffic congestions. In a situation where all vehicles slow down and reduce their speed by the same factor, the coefficient of variation should remain unchanged. However, observations show that this is not the case, and speed distribution changes with changes in the traffic conditions. From measurements of real vehicles speeds, taken by the Washington State Department of Transportation [23], it can be easily seen that the variation in actual velocities increases dramatically at rush hours, together with the number of vehicles (Fig. 4). This change reflects changes in the distribution of desired velocities, and indicates that drivers may act differently under different conditions. The different responses change the desired speed distribution and affect the congestion.

According to the U.S. Federal Highway Administration, 15% of the congestions are caused by weather [24]. Bad weather may cause congestion by many indirect ways—causing floods and accidents, opening pits, and providing other reasons for lane closures. In addition, the larger headway that is kept between vehicles decreases the effective road capacity and may take it to a lower level than the demand [25], causing traffic jams and congestions. However, every driver knows from self-experience that congestions are prevalent under bad weather conditions even without any special events and they may come and disappear without any observed reason. Our results suggest that a possible contribution to congestion formation comes from a change in the

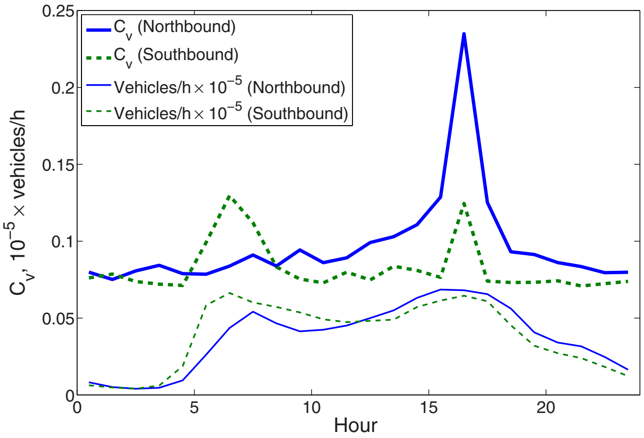

FIG. 4. (Color online) Number of vehicles (thin lines) and coefficient of variation of speed distribution (thick lines) at each time of the day. Measurements were taken by the Washington State Department of Transportation on May 27, 2008, at the interstate road I-5, near mile post 185. There are four lanes to each direction and the legal speed limit is 60 mph (about 96 kmh).

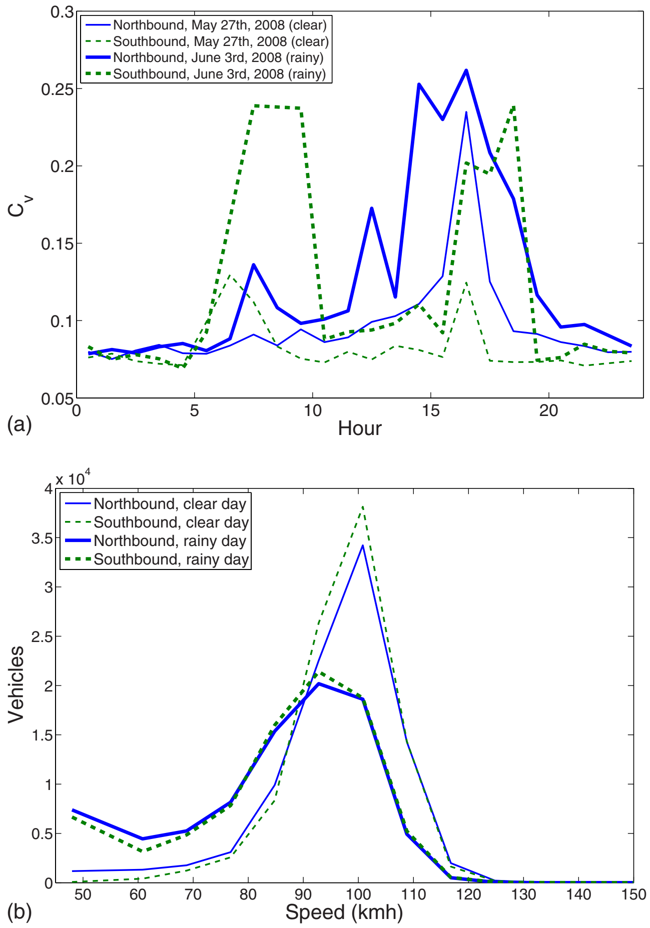

FIG. 5. (Color online) Comparison of speed distributions at clear and rainy days. (a) Coefficient of variation. In clear days (thin lines) there is high variation at rush hours only, whereas in a rainy day (thick lines) the variation is higher and may occur at any time of the day. Both measurements were taken at the same location, on two successive Tuesdays. (b) Speed distribution for the whole day. The total number of vehicles was similar in two days 90 571 (92 833) vehicles went northbound (southbound) on May 27, and 84 913 (84 456) on June 3, but the distribution is different. Weather information was taken from the National Oceanic and Atmospheric Administration [35].

distribution. The rate of slowing down is different from one driver to another—some drivers do not change their speed whereas others decrease it significantly. As a result, the speed distribution gets broader [26], and this change decreases the traffic flow efficiency. To verify this hypothesis, we have compared the speed distribution of a sunny day to that of a rainy one. Speed measurements were taken at the same location and at the same day of the week, and the total number of vehicles was similar. As shown in Fig. 5, the variation under rain condition is much higher and lasts for more hours than in a clear day. Part of it is due to the increasing portion of vehicles that are in jammed phase, with actual velocity close to zero. However, Fig. 5(B) shows that this is not the whole explanation. The speed distribution of the freely moving vehicles is broader in the rainy day than in the clear day. The wide distribution contributes to the high frequency of traffic congestions and significant slow down under bad weather conditions, even in cases there is no direct obstacle.

This observation suggests a simple method for improving traffic flow under conditions which are known to cause wide velocity distribution. All that it takes is dividing a multilane highway into groups of 2 or 3 lanes each, and directing vehicles to a specific group based on their speed. By doing so, we effectively create several parallel mini highways with narrow distribution in each of them. Since we showed above that the number of lanes is not a major parameter, the increase in efficiency due to narrowing the distribution is higher than the loss due to the reduction in number of lanes.

IV. DISCUSSION

When modeling collective behavior, such as traffic flow, evolution of populations, or financial dynamics, we often look for the balance between two opposing arguments. On the one hand, we tend to view the global variables as reliable descriptions not only of the population but also of the individuals within the population—the mean behavior is the behavior of the average individual. This is a simplifying argument which helps developing and analyzing models of complex systems. On the other hand, these models may miss important aspects of the system which are either driven by the intrapopulation variability and diversity or simply averaged out and can be viewed only in local observations [27,28]. It was Albert Einstein who stated that "It can scarcely be denied that the supreme goal of all theory is to make the irreducible basic elements as simple and as few as possible without having to surrender the adequate representation of a single datum of experience" [29]. This statement is usually quoted as "A theory should be as simple as possible, but no simpler." The same can be said about computational modeling of complex systems. A good model should include as few assumptions, constraints, or details as possible, and yet be able to capture and simulate the essence of the real system. This is the reason why, when possible, many modelers ignore the existence of variability in their parameters and refer to an average value only. Such models are valid and they can describe well many systems and provide useful insights. However, we should bear in mind that there are phenomena that will be missed by these models. In cases where the deviation from the average is the driving force for quantitative and qualitative phenomena, a more detailed model is required. Ignoring the variability in these cases may result in misleading conclusions. Variability was shown as a determinant factor in ecologic systems [30], neurology [31,32], social studies [33], and other fields [34]. Here we show that it plays a role in traffic management as well, and should be taken into account together when accurate prediction is needed.

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